Trade Dynamics with Endogenous Contact Rate

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Abstract

Afonso and Lagos (2015) introduced in trade dynamics model under fixed contact rate during the last 2.5 hours of trading session. This exercise is to study their work allowing endogenous contact rate.

1 Introduction

Financial institutions in US keep reserve balances at Federal Reserve Banks to meet requirements, earn interest, or clear financial transactions. This market allows institutions with excess reserve balances to lend reserves to institutions with reserve deficiencies. The fed funds market is primarily a mechanism that reallocates reserves among banks. It is a crucial market from the standpoint of the economics of payments and the branch of banking theory that studies the role of interbank markets in helping banks manage reserves and offset liquidity or payment shocks. The fed funds market is also the setting where the interest rate on the shortest maturity, most liquid instrument in the term structure is determined. This makes it an important market from the standpoint of finance.

2 Previous Literatures

In the field of studying fed funds market, Poole (1968), Ho and Saunders (1985), and Coleman, Gilles, and Labadie (1996) contributed in theoretical perspective whereas Hamlton (1996) and Hamlton and Jorda (2002) contributed in empirical perspective. Ashcraft and Duffie (2007) highlighted the over-the-counter nature of the fed funds market. Bech and Klee (2011), Ennis and Weinberg (2009), and Furfine (2003) used this nature to try to explain certain aspects of interbank markets such as apparent limits to arbitrage, stigma, and banks' decisions to borrow from standing facilities. Afonso and Lagos (2015) contributed a model to the intraday allocation of reserves and pricing of overnight loans using a dynamic equilibrium search-theoretic framework such that it can capture the salient features of the decentralized interbank market in which these loans are traded. Their work arises from emerging literature that studies search and bargaining frictions in financial markets.

3 Trade Dynamics

3.1 Benchmark Model

The population of banks, represented by a point in the interval [0,1]. During trading session, the reserve balances set in continuous time that starts at time 0 and ends at time T. Let τ denote the time remaining until the end of trading session, so $\tau = T - t$ if the current time is $t \in [0, T]$. The reserve balance at time $T - \tau$ is denoted by $k(\tau) \in \mathbb{K}$, with $\mathbb{K} = \{0, 1, ..., K\}$, where $K \in \mathbb{Z}$ and $1 \leq K$. The measure of banks with balance k at time $t - \tau$ is denoted $n_k(\tau)$. A bank starts the trading session with some balance $k(T) \in \mathbb{K}$. The initial distribution of balances, $\{n_k(T)\}_{k \in \mathbb{K}}$, is given. Let $u_k \in \mathbb{R}$ denote the flow payoff to a bank from holding k balances during the trading session, and let $U_k \in \mathbb{R}$ be the payoff from holding k balances at the end of the trading session. All banks discount payoffs at rate r.

Afonso and Lagos (2015) assume that loan sizes are elements of the set $\bar{\mathbb{K}} = \mathbb{K} \cup \{-K, ..., -1\}$ and that every loan gets repaid at time $T + \triangle$ in the following trading day, where $\triangle \in \mathbb{R}_+$. Let $x \in \mathbb{R}$ denote the net credit position that has resulted from some history of trades. Afonso and Lagos (2015) also assume that the payoff to a bank with a net credit position x that makes a new loan at time $T - \tau$ with repayment R at time $T + \triangle$ is equal to the post-transaction discounted net credit position, $e^{-r(\tau + \triangle)}(x + R)$.

Afonso and Lagos (2015) abstract brokers from the baseline model since nonbrokered transactions represent the majority of the volume. The search entails two layers of uncertainty in this environment. First, the time it takes a bank to contact a counterparty is an exponentially distributed random variable with mean $1/\alpha$. Second, conditional on having contacted a counterparty at time $T - \tau$, the reserve balance k of the counterparty is a random variable with probability distribution $\{n_k(\tau)\}_{k \in \mathbb{K}}$.

Let the function $V_k(\tau) : \mathbb{K} \times [0, T] \to R$ denote the maximum attainable payoff that a bank can obtain from $k \in \mathbb{K}$ units of reserve balances when the time until the end of the trading session is $\tau \in [0, T]$. During the trading hours, two banks meet and bargain over size of the loan and repayment. Consider a bank with k balances that contacts a bank with k' balances. For any pair of pre-trade reserve balances of two banks, $k, k' \in \mathbb{K}$, the set $\Pi(k, k') = \{(q, q') \in \mathbb{K} \times \mathbb{K} : q + q' = k + k'\}$ contains all feasible pairs of post-trade balances that could result from the bilateral negotiation. For every pair of banks that hold pre-trade balances $(k, k') \in \mathbb{K} \times \mathbb{K}$, the set $\Pi(k, k')$ induces the set of all feasible loan sizes, $\Gamma(k, k') = \{b \in \bar{\mathbb{K}} : (k - b, k' + b) \in \Pi(k, k')\}$. Notice that $\Pi(k, k') = \Pi(k', k)$ and $\Gamma(k, k') = -\Gamma(k', k)$ for all $k, k' \in \mathbb{K}$. Then we have $b_{kk'}(\tau)$ to be the amount of reserves that the bank with k lends to the bank with balance k', and $R_{k'k}(\tau)$ to be the amount of balances that the latter commits to repay the former at time $T + \triangle$. Thus the pair $(b_{kk'}(\tau), R_{k'k}(\tau))$ denotes the bilateral terms of trade between a bank with balance k and a bank with balance k' when the remaining time until the end of the trading session is τ. These terms of trade will be the outcome corresponding to the symmetric Nash solution to the bilateral bargaining problem. Then, for any $k, k' \in \mathbb{K}$ and any $\tau \in [0, T]$, $(b_{kk'}(\tau), R_{k'k}(\tau))$ is the solution to

$$\max_{b \in \Gamma(k,k'), R \in \mathbb{R}} [V_{k-b}(\tau) + e^{-r(\tau+\triangle)} R - V_k(\tau)] \times [V_{k'+b}(\tau) - e^{-r(\tau+\triangle)} R - V_{k'}(\tau)].$$

Thus, for any $k \in \mathbb{K}$ and any $\tau \in [0, T]$,

$$V_k(\tau) = \mathbb{E}\Bigg\{ \int_0^{min(\tau_\alpha, \tau)} e^{-rz} u_k dz + \mathbb{I}_{\{\tau_\alpha > \tau\}} e^{-r\tau} U_k +$$

$$\mathbb{I}_{\{\tau_\alpha \le \tau\}} e^{-r\tau_\alpha} \sum_{k' \in K} n_{k'}(\tau - \tau_\alpha) \times [V_{k - b_{kk'}(\tau - \tau_\alpha)}(\tau - \tau_\alpha) + e^{-r(\tau + \triangle - \tau_\alpha)} R_{k'k}(\tau - \tau_\alpha)] \Bigg\} \quad (1)$$

where

$$b_{kk'}(\tau) \in arg \max_{b \in \Gamma(k,k')} [V_{k'+b}(\tau) + V_{k-b}(\tau) - V_{k'}(\tau) - V_k(\tau)], \quad (2)$$

$$e^{-r(\tau+\triangle)} R_{k'k}(\tau) = \frac{1}{2}[V_{k' + b_{kk'}(\tau)}(\tau) - V_{k'}(\tau)] + \frac{1}{2}[V_k(\tau) - V_{k - b_{kk'}(\tau)}(\tau)]. \quad (3)$$

The expectation operator, $\mathbb{E}$, in (1) is with respect to the exponentially distributed random time until the next trading opportunity, τ_α, and $\mathbb{I}_{\tau_\alpha \le \tau}$ is an indicator function that equals 1 if $\tau_\alpha \le \tau$ and 0 otherwise. The first term contains the flow payoff to the bank from holding balance k until the next trade opportunity or the end of the session, whichever arrives first. The second term says that in the event that the bank gets no trading opportunity before time T. The third term contains the expected discounted payoff if the event that the bank gets a trade opportunity with another bank before time T, that is, at time $T - (\tau - \tau_\alpha)$. In this event, the counterparty is a random draw from the distribution of balances at time $T - (\tau - \tau_\alpha)$, namely, $\{n_{k'}(\tau - \tau_\alpha)\}_{k' \in \mathbb{K}}$, and the expression inside the square bracket is the post-trade continuation payoff of the bank we are considering. Hereafter, Afonso and Lagos (2015) use $\mathbf{V} \equiv [\mathbf{V}(\tau)]_{\tau\in[0,T]}$, with $\mathbf{V}(\tau) \equiv \{V_k(\tau)\}_{k\in\mathbb{K}}$ to denote the value function. According to the bargaining solution (2) and (3), the loan size maximizes the joint gain from trade, and the repayment implements a division of this gain between the borrower and the lender that gives each a fraction equal to their bargaining power (that is, one half). For example, if a bank with $i \in \mathbb{K}$ balances and a bank with $j \in \mathbb{K}$ balances meet at time $T-\tau$, they will negotiate a loan of size $b_{ij}(\tau) = i - k = s - j$, where $(k,s) \in arg\ max_{(k',s')\in\Pi(i,j)} S_{ij}^{k's'}(\tau) \equiv V_{k'}(\tau) + V_{s'}(\tau) - V_i(\tau) - V_j(\tau)$. The implied joint gain from trade corresponding to this transaction is $S_{ij}^{ks}(\tau)$.

Consider a bank with i balances that contracts a bank with j balances when the time until the end of trading session is τ. Let $\phi_{ij}^{ks}(\tau)$ be the probability that the former and the latter hold k and s balances after the meeting, respectively, that is, $\phi_{ij}^{ks}(\tau) \in [0,1]$, with $\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}} \phi_{ij}^{ks}(\tau) = 1$. Feasibility requires that $\phi_{ij}^{ks}(\tau) = 0$ if $(k,s) \notin \Pi(i,j)$. Given any feasible path for the distribution of trading probabilities, $\phi(\tau) = \{\phi_{ij}^{ks}(\tau)\}_{i,j,k,s,\in\mathbb{K}}$, the distribution of balances at time $T-\tau$, that is, $n(\tau) = \{n_k(\tau)\}_{n\in\mathbb{K}}$, evolves according to

$$\dot{n}_k(\tau) = f[\mathbf{n}(\tau), \phi(\tau)]\ \forall k \in \mathbb{K}, \tag{4}$$

where

$$f[\mathbf{n}(\tau),\phi(\tau)] \equiv \alpha n_k(\tau) \sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_i(\tau)\phi_{ki}^{sj}(\tau) - \alpha \sum_{i\in\mathbb{K}}\sum_{j\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_i(\tau) n_j(\tau)\phi_{ij}^{ks}(\tau). \tag{5}$$

The first term on the right side of (5) contains the total flow of banks that leave state$k$ between time $t = T-\tau$ and time $t' = T - (\tau - \epsilon)$ for a small $\epsilon > 0$. The second term contains the total flow of banks into state k over the same interval of time.

The following proposition provides a sharper representation of the value function and the distribution of trading probabilities characterized in (1), (2), and (3).

PROPOSITION 1: *The value function* $\mathbf{V}$ *satisfies (1), with (2) and (3), if and only if it satisfies*

$$rV_i(\tau)+\dot{V}_i(\tau) = u_i + \frac{\alpha}{2}\sum_{j\in\mathbb{K}}\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_j(\tau)\phi_{ij}^{ks}(\tau)\times[V_k(\tau)+V_s(\tau)-V_i(\tau)-V_j(\tau)]\ \forall(i,\tau) \in \mathbb{K}\times[0,T] \tag{6}$$

with

$$V_i(0) = U_i\ \forall i \in \mathbb{K} \tag{7}$$

and

$$\phi_{ij}^{ks}(\tau)\begin{cases} \leq 0 \;\; if \;\; (k,s)\in\Omega_{ij}[\mathbf{V}(\tau)] \\ =0 \;\; if \;\; (k,s)\notin\Omega_{ij}[\mathbf{V}(\tau)] \end{cases}, \forall i,j,k,s \in \mathbb{K}, with \sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}}\phi_{ij}^{ks}(\tau) = 1, \tag{8}$$

where

$$\Omega_{ij}[\mathbf{V}(\tau)] \equiv arg \max_{(k',s')\in\Pi(i,j)} [V_{k'}(\tau) + V_{s'}(\tau) - V_i(\tau) - V_j(\tau)] \tag{9}$$

The set $\Omega_{ij}[\mathbf{V}(\tau)]$ defined in (9) contains all the feasible pairs of post-trade balances that maximize the joint gain from trade between a bank with i balances and a bank with j balances that is implied by the value function $\mathbf{V}(\tau)$ at time $T-\tau$. For any pair of banks with balances i and j, $\phi_{ij}^{ks}(\tau)$ defined in (8) is probability distribution over the feasible pairs of post-trade balances that maximize the bilateral gain from trade. Thus, (8) and (9) together describe the pairs of post-trade balances (or equivalently, loan sizes) that may result from the bilateral bargaining. The Bellman equation described by (6) and (7) has a natural interpretation. The flow value of a bank that holds balance i at time $T-\tau$, that is, $rV_i(\tau)$, consists of the flow return from holding balance i, that is, u_i, minus the flow capital loss associated with the reduction in the remaining time until the end of the trading session, that is, $\dot{V}_i(\tau)$, plus the rate at which the bank meets counterparties, α, times the expected gain from trade to the bank, that is, $\sum_{j\in\mathbb{K}}\sum_{k\in\mathbb{K}}\sum_{s\in\mathbb{K}} n_j(\tau)\phi_{ij}^{js}(\tau)\frac{1}{2}S_{ij}^{ks}(\tau)$.

Definition 1: An equilibrium is a path for the distribution of reserve balances, $\mathbf{n}(\tau)$, a value function, $\mathbf{V}$, and a path for the distribution of trading probabilities, $\phi(\tau)$, such that, (a) given the value function and the distribution of trading probabilities, the distribution of balances evolves according to (4); and (b) given the path for the distribution of balances, the value function satisfies (6) and (7), and the distribution of trading probabilities satisfies (8).

In quantitative implementations of the theory, one can try to compute equilibrium algorithmically, as follows. Guess a path of trading probabilities and use it to solve the system of differential equations (4) with initial condition $\{n_k(T)\}_{k\in\mathbb{K}}$ to obtain a path for the distribution of reserve balances. Then substitute the trading probabilities and distribution of reserves implied by the guess, and solve the system of differential equations (6) and (7) for the implied value function. Then use this value function and (8) to obtain a new guess for the path of trading probabilities and continue iterating until the value function has converged. Instead of following this route, Afonso and Lagos (2015) make a curvature assumption on the vectors$\{u_k\}_{k\in\mathbb{K}}$ and $\{U_k\}_{k\in\mathbb{K}}$ that will allow us to provide an analytical characterization of equilibrium.

ASSUMPTION A: $\forall i,j\in\mathbb{K}$, *the payoff functions satisfy*

$$(DMC)\ u_{\lceil (i+j)/2\rceil}+u_{\lfloor (i+j)/2\rfloor}\leq u_i+u_j,$$

$$(DMSC)\ U_{\lceil (i+j)/2\rceil}+U_{\lfloor (i+j)/2\rfloor}\leq U_i+U_j,$$
$$\text{“}>\text{''}\ \ unless\ i=\left\lceil\frac{i+j}{2}\right\rceil\ and\ j=\left\lfloor\frac{i+j}{2}\right\rfloor,$$
$$where\ \lfloor x\rfloor\equiv\max\{k\in\mathbb{Z}:k\leq x\}\ and\ \lceil x\rceil\equiv min\{k\in\mathbb{Z}:x\leq k\}\forall x\in\mathbb{R}.$$

Conditions (DMC) and (DMSC) require that $\{u_k\}_{k\in\mathbb{K}}$ and $\{U_k\}_{k\in\mathbb{K}}$ satisfy the *discrete midpoint concavity property* and the *discrete midpoint strict concavity property*, respectively. These conditions are the natural discrete approximations to the notions of *midpoint concavity* and *midpoint strict concavity* of ordinary functions defined on convex sets. Assumption A is reasonable in the context of the fed funds market because central banks typically do not offer payment schemes that are convex in reserve balances. The following result provides a full characterization of equilibrium under this assumption.

PROPOSITION 2: *Let the payoff functions satisfy Assumption A. Then:*

(i) *An equilibrium exists, and the equilibrium paths for the maximum attainable payoffs,* $\mathbf{V}(\tau)$*, and the distribution of reserve balances,* $\mathbf{n}(\tau)$*, are uniquely determined.*

(ii) *The equilibrium path for the distribution of trading probabilities,* $\phi(\tau)=\{\phi_{ij}^{ks}(\tau\}_{i,j,k,s,\in\mathbb{K}}$*, is given by*

$$\phi_{ij}^{ks}(\tau)\begin{cases}\leq 0\ if\ (k,s)\in\Omega_{ij}^{*}\\ =0\ if\ (k,s)\notin\Omega_{ij}^{*}\end{cases},\forall i,j,k,s\in\mathbb{K}\ and\ \forall\tau\in[0,T],\ with\sum_{(k,s)\in\Omega_{ij}^{*}}(\tau)=1,and \tag{10}$$

$$\Omega_{ij}^{*}=\begin{cases}\left\{\left(\frac{i+j}{2}\,\frac{i+j}{2}\right)\right\}\ if\ i+j\ is\ even,\\ \left\{\left(\left\lfloor\frac{i+j}{2}\right\rfloor,\left\lceil\frac{i+j}{2}\right\rceil\right),\left(\left\lceil\frac{i+j}{2}\right\rceil,\left\lfloor\frac{i+j}{2}\right\rfloor\right)\right\}\ if\ i+j\ is\ odd.\end{cases} \tag{11}$$

Murota (2003) provide detailed information on the midpoint concavity property and the role that it plays in the modern theory of discrete convex analysis. Let X be a convex subset of $\mathbb{R}^n$; then a function $g:X\to\mathbb{R}$ is *midpoint concave* if $2g(\frac{x+y}{2})\geq g(x)+g(y)$ for all $x,y\in X$. Clearly, if g is concave, it is midpoint concave. The converse is true provided g is continuous. The equilibrium distribution of trading probabilities (10) can be described intuitively as follows. If at any point during the trading session, a bank with balance i contacts a bank with balance j, then the post-trade balance is $\lfloor \frac{i+j}{2} \rfloor$ for one of the banks and $\lceil \frac{i+j}{2} \rceil$ for the other. This property, and the uniqueness of the equilibrium paths for the distribution of reserve balances and mximum payoffs, hold under Assumption A. With the path for $\phi(\tau)$ given by (10) in closed form, the equilibrium value function, $\mathbf{V}$, is the unique bounded real-valued function that satisfies (6) and (7), and the path for the distribution of balances, $\mathbf{n}(\tau)$, is given by (4) with initial condition $\{n_k(T)\}_{n\in\mathbb{K}}$.

3.2 Endogenous Contact Rate

Assume contact rate to be SOMETHING. Contact rate follow the characteristics of bell-shape distribution with a mean at the middle of the last 2.5 hours of trading session.[1]

4 Simulation

Afonso and Lagos (2015) selects time in days. The model is meant to capture trade dynamics in the last 2.5 hours of the daily trading session, so the model sets $T = 2.5/24$. Their work adopt the following banks' end-of-day payoffs:

$$U_k = e^{er\triangle_f}(k - \bar{k}_0 + F_k)$$

where

$$F_k = max_{k^w\in\mathbb{K}}\{\bar{F}(k^w) - i_f^w[k^w - (k - \bar{k}_0)]\}\ s.t.\ k - \bar{k}_0 \leq k^w$$

with

$$\bar{F}(k^w) = \begin{cases} i_f^r\bar{k} + i_f^e(k^w - \bar{k}) & \text{if } \bar{k} < k^w \\ i_f^r k^w - i_f^c(\bar{k} - k^w) & \text{if } 0 \leq k < \bar{k} \\ -i_f^c\bar{k} + i_f^o k^w & \text{if } k^w < 0 \end{cases}$$

The flow payoff to a bank from holding intraday balances is given by

$$u_k = \begin{cases} i_+^d(k - \bar{k}_0)^{1-\epsilon} & \text{if } 0 \leq k - \bar{k}_0 \\ i_-^d(\bar{k}_0 - k)^{1+\epsilon} & \text{if } k - \bar{k}_0 < 0 \end{cases}$$

[1] I have not finished this part yet. I always end up assuming conclusions. The challenge is to assume contact rate to be something and it has a bell-shape characteristic.

The parameterization of the initial distribution of reserve balances, $\{n_k(T)\}_{k\in K}$, is guided by identifying $n_k(T)$ in the theory with the empirical proportion of commercial banks whose balances at 4:00 pm are $k/\bar{k}$ times their average daily reserve requirement over a typical two-week holding period in 2007. The distribution of average imputed normalized reserve balances across the sample of 134 banks had a mean equal to 3.24. Afonso and Lagos (2015) translated this distribution so that the mean would match the empirical mean of the ratio of seasonally adjusted reserves to required reserves of all depository institutions during the second quarter of 2007, that is, 1.04 as reported in the H.3 Federal Reserve Statistical Release. Let h^i denote this average imputed normalized translated reserve balance for bank i. They let $\mathbb{K} = \{0, ..., 250\}$, $\bar{k}_0 = 100$, and set $n_k(T) = \frac{1}{134}\sum_i \mathbb{I}_{h^i\in[k-\bar{k}_0,k-\bar{k}_0+1]}$ for $k = 0, ..., 249$, and $n_{250}(T)1 - \sum_{k=0}^{249} n_k(T)$. Then they normalize $\bar{k} = 1$, so k can be interpreted as a multiple of the reserve requirement.

Most transactions are settled through Fedwire, and Fedwire does not operate for 2.5 hours 6:30 pm and 9:00 pm, so the settlement lags $\triangle$ and $\triangle_f$ are set to $\triangle = \triangle_f = 2.5/24$, which means that banks regard all loans (to other banks or to the Fed) as being repaid at the beginning of the following working day. The values of the policy rates i^d_-, i^d_+, i^r_f, i^e_f, i^c_f, and i^o_f are chosen to mimic policies in the United States during the second quarter of 2007. The interest rate charged on daylight overdrafts, i^d_-, is set to 0.0036/360. The interest rate paid on positive intraday balances, i^d_+, is set to $10^{-7}/360$. The Federal Reserve did not pay interest on reserves prior to October 2008, so $i^r_f = i^e_f = 0$. The total per-dollar cost of borrowing from the discount window is $i^r_f = i^e_f = 0$. The total per-dollar cost of borrowing from the discount window is $i^w_f = i^w + P^w$, where i^w is the window discount rate and P^w represents the pecuniary value of the additional costs associated with discount window borrowing. The deficiency charge for failing to meet the reserve requirement is $i^c_f = i^c + P^c$, where i^c is the overnight interest rate charged on the shortfall and P^c represents the pecuniary value of additional penalties that the bank may suffer for failing to meet reserve requirements. The overnight overdraft penalty rate is $i^o_f = i^o + P^o$, where i^o is the interest rate on the overdraft and P^o represents additional penalties resulting from the use of unauthorized overnight credit. The interest rate on discount window loans under the Primary Credit Facility was 6.25 percent per annum in the second quarter of

2007, so we set $i^w = 0.0625/360$. The penalty rate charged for reserve deficiencies is 100 basis points above the Primary Credit Facility discount window lending rate on the first day of the calendar month in which the deficiency occurred, so we set $i^c = 0.0725/360$. The interest penalty on overnight overdrafts is 400 basis points above the effective fed funds rate. The average daily effective fed funds rate during the second quarter of 2007 was 5.25 percent per annum, so we set $i^o = 0.0925/360$. In the baseline, Afonso and Lagos (2015) set $P^c = P^o = P^w$. We calibrate P^w and α, so that the equilibrium of the model is consistent with the following two targets: (a) the fed funds rate during the second quarter of 2007, which was 0.0525 per annum, and (b) the standard deviation of the empirical end-of-day distribution of average normalized reserve balances (for the two-week holding period used to estimate the initial distribution), which was 0.92. This calibration strategy implies that $P^w = 0.0525/360$ and $\alpha = 120$[2].

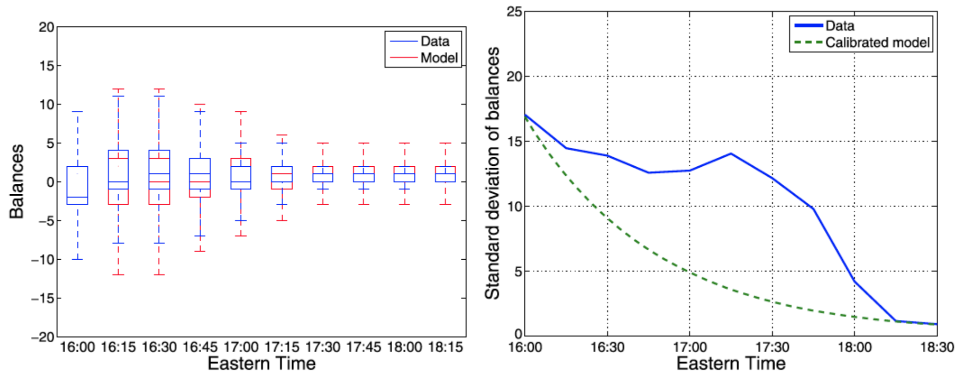
4.1 Afonso and Lagos (2015) Laboratory Result

We take Afonso and Lagos (2015) laboratory results as given (see Graph 1). We take Afonso and Lagos (2015) data as given and assumed original data including initial balance distribution for a million banks accordingly.

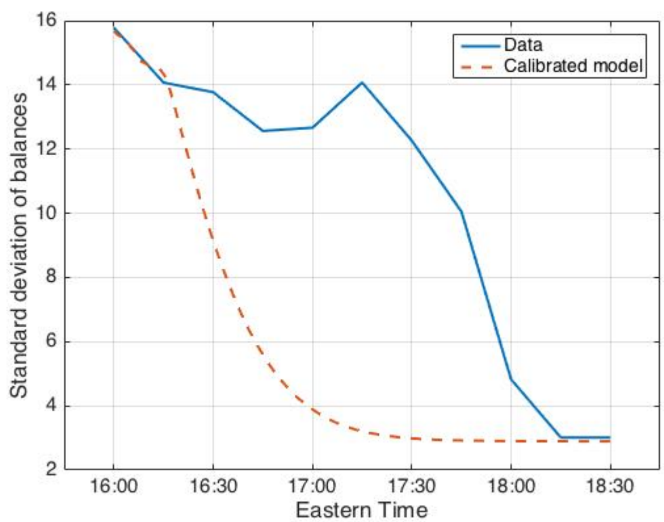
We solve value iteration function, equation (1), and find trading probability, ϕ, for distribution balance. Given the initial distribution balance, we can use equation (4), $\dot{n}_k(\tau)$ to find the distribution balance at the next time period, assuming 15-min box plot. We have the replicated standard deviation to be presented in Graph 2.

Graph 1. This is the balance distribution (left panel) and standard deviation (right panel) from Afonso and Lagos (2015). [3]

[2]Afonso and Lagos (2015) calibrate α such that the two targets about fed funds rate and standard deviation can be satisfied. The value $\alpha = 120$ implies that banks have an average of about 12.5 meetings during the trading session, that is, a trading opportunity every 12 minutes, on average.

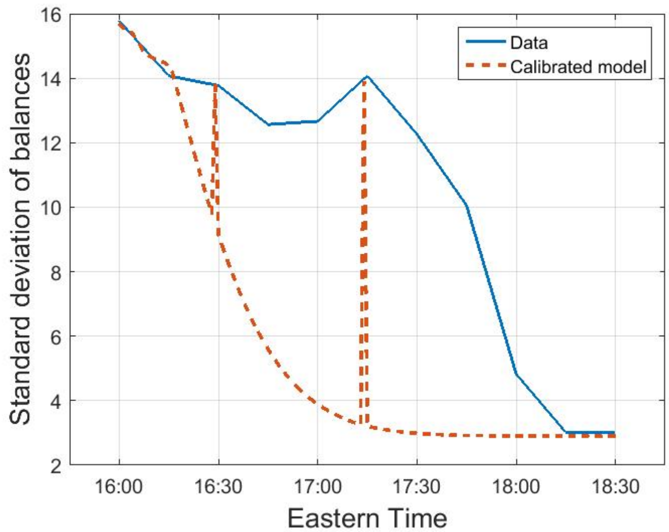


Graph 2. This is our replication of the right panel of Graph 1.



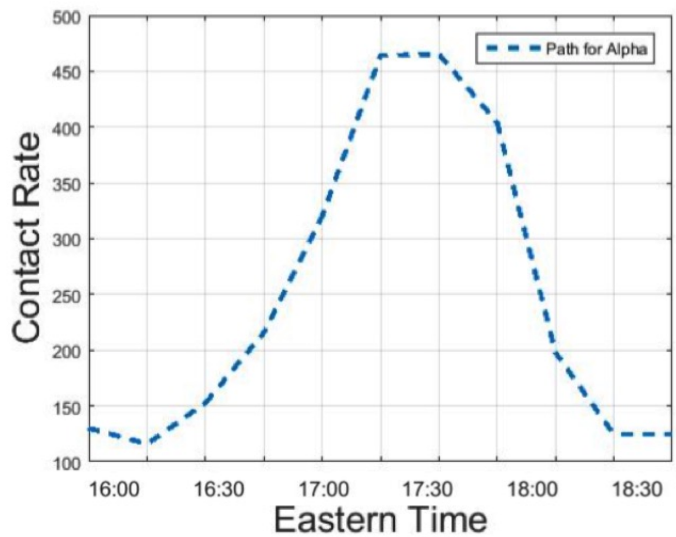
We compare the simulated model and the data. At 4:30PM and 5:15PM, there were two spikes in the data. We run simulation, in minute plot, to the selected time node and find an alpha so that the standard deviation match the data. We can complete these actions for the two "spikes" in the data, presented in Graph 3.

Graph 3. We can chose the time for particular spikes in the data and select contact rates, alphas, to match the data in the model.



We can apply the same process for all of the time nodes in the data. We can push the entire curve of standard deviation for the model up to match the curve of standard deviation for the data. To do so, we will have an endogenous contact rate, α, over time and we plot this path for alpha in Graph 4.

Graph 4. We can extract all points from data and select contact rates, alphas, to force the model to match the data. We plot the alpha path over time that suffice this match.



5 Conclusion

This exercise studies Afonso and Lagos (2015) work on trade dynamics in fed funds market. The work presents a laboratory environment for higher level studies to under federal fund markets and fund transactions. The purpose for this exercise is to allow contact rate to be endogenous. Allowing contact rate to change over time, we have found the exact alpha such that the model can match the data.

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